Control Lec 10

→ Transformation from general form of state space to another form :-

$$\boxed{\begin{array}{l}\dot{x} = Ax + Bu \\ y = Cx\end{array}} \xrightarrow[\text{Linear Trans.}]{x = TZ} \boxed{\begin{array}{l}\dot{Z} = \bar{A}Z + \bar{B}u \\ y = \bar{C}Z\end{array}}$$

"original form" "new form"

ـــ مجرد تحويلة رياضية ولكنها نفس ال T.F ونفس ال Poles و zeros .

Given: A, B, C

find T ⇒ transfomation matrix $\bar{A}, \bar{B}, \bar{C}$ $(\text{square})_{n \times n}$

$x(t) = T\,Z(t)$ (Linear transformation)

n ـــ درجة النظام

$\dot{x} = T\dot{Z}$

$\dot{x} = Ax + Bu$

$T\dot{Z} = ATZ + Bu$

$\dot{Z} = T^{-1}ATZ + T^{-1}Bu$

$\bar{A} = T^{-1}AT$

$\bar{B} = T^{-1}B$

تطلع المعاملات بالشكل ده.

old form

$$\boxed{\begin{matrix}\dot{x} = Ax + Bu \\ y = Cx\end{matrix}} \xrightarrow{X = TZ} \boxed{\begin{matrix}\dot{z} = \bar{A}Z + \bar{B}u \\ y = \bar{C}Z\end{matrix}}$$

new form

ـــ غالباً ما يتم التحويل من صورة ال (system) إلى (Diagonal)
أو (Controllable)

ـــ التحويل يتم إلى صورة ال (Controllable) تم دراسته.

"Transformation to Diagonal Form"

Assume 2nd order system

$S_2, S_1 \longrightarrow$ Poles

$$\bar{A} = T^{-1} A T = \begin{pmatrix} S_1 & 0 \\ 0 & S_2 \end{pmatrix}$$

$$T = (V_1 \quad V_2) \; , \; V_1 = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} \; , \; V_2 = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

ـــ نطلع ال I بواسطة ①، ② حسب درجة ال sys.

$$\boxed{(SI - A) V_i = 0 \qquad ; \; i = 1, 2}$$

$$(S_1 I - A) V_1 = 0 \qquad ; \qquad (S_2 I - A) V_2 = 0$$

$$\begin{pmatrix} S_1 & 0 \\ 0 & S_2 \end{pmatrix} = T^{-1} A T$$

$$T \bar{A} = A T$$

$$T\begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix} = AT$$

$$\begin{pmatrix} V_1 & V_2 \end{pmatrix}\begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix} = AT$$

<u>Example</u>

$$\dot{x} = \begin{pmatrix} -5 & -1 \\ 3 & -1 \end{pmatrix} x + \begin{pmatrix} 2 \\ 5 \end{pmatrix} u \quad ; \quad y = \begin{pmatrix} 1 & 2 \end{pmatrix} x$$

ال chls وال (Poles) كما هي في الشكل القديم لل (sys.) والجديد.

Poles $\Rightarrow |sI - A| = 0$

$$\left| \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} -5 & -1 \\ 3 & -1 \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} s+5 & (-)\,1 \\ -3 & s+1 \end{vmatrix} = 0 \Rightarrow (s+5)(s+1) + 3 = 0$$

$$\underset{\downarrow i}{s^2} + 6\underset{\downarrow i}{s} + 8 = 0$$

for $I = 1 \Rightarrow s = -2$ first pole

$$(s_i - A)\, V_i = 0$$

$$\begin{pmatrix} S_i + 5 & 1 \\ -3 & S_i + 1 \end{pmatrix} V_i = 0$$

$$i = 1 \Rightarrow \begin{pmatrix} S_1 + 5 & 1 \\ -3 & S_1 + 1 \end{pmatrix}$$

$$\begin{pmatrix} -2 + 5 & 1 \\ -3 & -2 + 1 \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = 0$$

نفس المعادلة لكن بالسالب

$$3a_1 + b_1 = 0 \quad ; \quad -3a_1 - b_1 = 0$$

وبالتالي هنفرض أول قيمة ب 1

$$a_1 = 1 \quad ; \quad b_1 = -3$$

$$\therefore V_1 = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$$\text{if } i = 2 \Rightarrow S_2 = -4 \quad \& \quad V_2 = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

$$\begin{pmatrix} S_2 + 5 & 1 \\ -3 & S_2 + 1 \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = 0$$

$$\begin{pmatrix} 1 & 1 \\ -3 & -3 \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = 0$$

$a_2 + b_2 = 0$ نفس المعادله لكن الثانية مضروبة في -3.

$-3a_2 - 3b_2 = 0$

assume $a_2 = 1 \Rightarrow b_2 = -1 \Rightarrow V_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$$T = \begin{pmatrix} V_1 & V_2 \end{pmatrix} = \begin{pmatrix} +1 & +1 \\ -3 & -1 \end{pmatrix}$$

$$T^{-1} = \frac{1}{|T|} \begin{pmatrix} -1 & -1 \\ 3 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 & -1 \\ 3 & 1 \end{pmatrix}$$

$$\bar{A} = T^{-1} A T$$

$$= \frac{1}{2} \begin{pmatrix} -1 & -1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} -5 & -1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -3 & -1 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} -1 & -1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} -2 & -4 \\ 6 & 4 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} -1 & -1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} -1 & -2 \\ 3 & 2 \end{pmatrix}$$

$$\bar{A} = \begin{pmatrix} -2 & 0 \\ 0 & -4 \end{pmatrix} \longrightarrow \text{system Poles.}$$

$$\bar{B} = T^{-1} B$$

$$= \frac{1}{2}\begin{pmatrix} -1 & -1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -7 \\ 11 \end{pmatrix} = \begin{pmatrix} -3.5 \\ 5.5 \end{pmatrix}$$

$$\bar{C} = CT = \begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -3 & -1 \end{pmatrix} = \begin{pmatrix} -5 & -1 \end{pmatrix}$$

$$\dot{Z} = \begin{pmatrix} -2 & 0 \\ 0 & -4 \end{pmatrix} Z + \begin{pmatrix} -3.5 \\ 5.5 \end{pmatrix} u$$

ال (System) له عدد لا نهائي من
ال (state equation)

$$y = \begin{pmatrix} -5 & -1 \end{pmatrix} Z$$

Special Case — Transformation Into Diagonal Form

$$x = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 2 \end{pmatrix} x$$

Poles $\Rightarrow$ Ch/s eqn $\Rightarrow |sI - A| = 0$

$$\left| \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & -1 \\ -2 & -3 \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} s & -1 \\ 2 & s+3 \end{vmatrix} = 0 \Rightarrow s^2 + 3s + 2 = 0 \rightsquigarrow \text{ch. eqn}$$

$$(s+1)(s+2) = 0$$

Thus Poles $\Rightarrow -1, -2$

$(sI - A) V_i = 0$

<u>for $i=1$</u>, $s_1 = -1$, $V_i = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$

$$\begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = 0$$

$-a_1 - b_1 = 0$, $2a_1 + 2b_1 = 0$

assume $a_1 = 1 \Rightarrow b_1 = -1$. $\therefore V_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

for $i=2$, $s_2 = -2$, $V_2 = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$

$$\begin{pmatrix} -2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = 0$$

$-2a_2 - b_2 = 0$

$2a_2 + b_2 = 0$

assume $a_2 = 1 \Rightarrow b_2 = -2$

$$\therefore V_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$T = (V_1 \;,\; V_2) = \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}$$

$$\bar{A} = T^{-1} A T = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$$

$$B = T^{-1} B = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad ; \bar{C} = CT = \begin{pmatrix} -1 & 3 \end{pmatrix}$$

← لو الـ (matrix) ذ الـ (Controllable) وعايز أحولها لـ (Diagonal) لها حالة خاصة.

Special Case: A in Controllable.

$$T = \begin{pmatrix} 1 & 1 \\ s_1 & s_2 \end{pmatrix} \quad ; s_1, s_2 \longrightarrow \text{Poles}$$

$$T = \begin{pmatrix} 1 & 1 & 1 \\ s_1 & s_2 & s_3 \\ s_1^2 & s_2^2 & s_3^2 \end{pmatrix} \quad ; s_1, s_2, s_3 \Rightarrow \text{Poles}$$

For Repeated Poles

← نترك العمود الأول كما هو ، والعمود الثانى هو تفاضل العمود الأول .

Poles → s_1, s_1, s_2

$$T = \begin{pmatrix} 1 & 1 & 1 \\ s_1 & s_1 & s_2 \\ s_1^2 & s_1^2 & s_2^2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 1 \\ s_1 & 1 & s_2 \\ s_1^2 & 2s_1 & s_2^2 \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 0 \\ s_1 & 1 \end{pmatrix} \qquad s_1, s_1 \longrightarrow \text{Poles}$$

هذا الـ (Transform) مش معانا في الإمتحان

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* state feedback Control / Pole Placement design

classifications :-

1) classical Control (PI, PD, PID, Phase(lead, lag))

2) modern Control (observer, state feedback)

3) Computational Artificial Control Design (neural, fuzzy, ...)

* modern Control design

→ The system model based on s.s (state space)

مهم ← جه فى الإمتحان قبل كده.

Control

1) servo Problem control / design

I/P ← ليها $(r \neq 0)$

regulation Problem design / Control

$(r = 0)$ → I/P Zero

(system) فى حالة error , noise
كبيرة عايز يتلاشاها فبالتالى يفحص وجود o/p
للـ sys. دون وجود I/P
حالة منها (state feed back Control)

* Regulation Problem design :-

when Control is done to reject (desired I/P), The o/P match this setting Point, this Process is called servo control.

* state Feedback Control

- regulation problem
- modern control design.

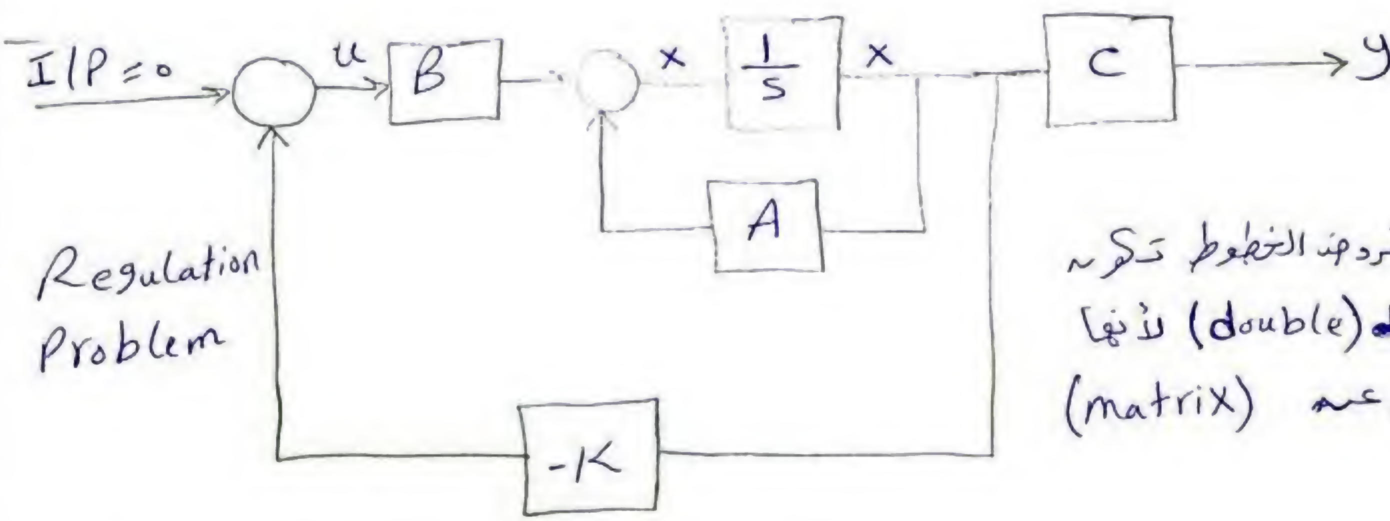
Given $\boxed{\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}}$

Control law :- $u = -Kx$

↳ Gain matrix

ـ بالشكل ده مفيش I/P و إعتبرت إن عندي (Disturbance)
اللي هو طلع o/p .

→ The design is to find the Gain K



ـ المفروض الخطوط تكون
(double) لأنها
تعبر عن (matrix)

* To find the Gain K

$\dot{x} = Ax + Bu$ $\qquad y = Cx$

$u = -Kx$

بالتعويض

$\dot{x} = Ax - BKx$

$= (A - BK)x$ → state eqn after using control loop

The desired chls equation

$|sI - (A - BK)| = 0 \Rightarrow (1)$

*Given design specs

The location of the desired poles

Assume 2nd order system

Given: P_1, P_2 (Poles)

The desired chls eqn $\alpha_c(s)$

$\alpha_c(s) = (s - P_1)(s - P_2) = 0 \longrightarrow (2)$

from (1) , (2) we can obtain the Gain K

$K = [K_1, K_2, \ldots\ldots, K_n]$

$n \rightarrow$ system order.

Example $\dot{x} = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t)$

$$y = \begin{pmatrix} 1 & -1 \end{pmatrix} x$$

* Design a state feedback Controller to Place (locate) the closed loop Poles at $S = -1 \pm J2$

① Desired chls eqn $\alpha_c(s)$:-

$$\alpha_c(s) = (\underbrace{s+1}_{x} - \underbrace{J2}_{y})(\underbrace{s+1}_{x} + \underbrace{J2}_{y}) = 0$$

$$\alpha_c(s) = (s-1)^2 + 4 = 0$$

$$\alpha_c(s) = s^2 - 2s + 5 = 0 \longrightarrow ①$$

② The desired chls eqn After using Control Law $u = -Kx$

$$|sI - (A - BK)| = 0$$

$$A - BK = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} K_1 & K_2 \end{pmatrix}$$

$$A - BK = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} K_1 & K_2 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 2-K_1 & -K_2 \\ 1 & 0 \end{pmatrix}$$

$$|sI - (A-BK)| = 0$$

$$\left| \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 2-K_1 & -K_2 \\ 1 & 0 \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} s-2+K_1 & K_2 \\ -1 & s \end{vmatrix} = 0$$

$$s^2 - 2s + K_1 s + K_2 = 0$$

$$s^2 + (K_1 - 2)s + K_2 = 0 \Rightarrow (2)$$ ch.s desired eqn.

ـــ بمقارنة المعاملات بمعادلة رقم (1)

$$s^2 - 2s + 5 = 0$$

$$K_1 - 2 = 2 \Rightarrow K_1 = 4 \quad , \quad K_2 = 5$$

$\boxed{\text{هام}}$

"check Controllability"

$$M_c = (B \quad AB)$$

ال (system) لازم يبقى (Controllable) عشان أعمل ال (design).

$$|M_c| \neq 0 \quad \text{controllable}$$

لازم أعمل (check) قبل الحل.

ـــ ممكن يجي في الإمتحان.

Another solution

*** Ackermann's method**

Replace $s \longrightarrow A$

$$K = [K_1 \quad K_2] = [0 \quad 1]\, M_c^{-1}\, \alpha_c(A)$$

$$\alpha_c(A) \Rightarrow \alpha_c(s)\Big|_{s=A}$$ Desired ch. eqn

$M_c \Rightarrow$ controllability matrix

*** In 3rd order**

$$K = [K_1 \quad K_2 \quad K_3] = (0 \quad 0 \quad 1)\, M_c^{-1}\, \alpha_c(A)$$

1) check controllability

$|M_c| \neq 0$ $\qquad M_c = (B \quad AB)$

$$M_c = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \Rightarrow M_c^{-1} = \begin{bmatrix} 1 & 2 * -1 \\ -1 * 0 & 1 \end{bmatrix} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$$

$\therefore |M_c| = 1 \neq 0$ (Controllable system)

$$K = \begin{pmatrix} 0 & 1 \end{pmatrix} M_c^{-1} \, \alpha_c(A)$$

$$M_c^{-1} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$$

(حسبناها من شوي)

$$\alpha_c(s) = s^2 + 2s + 5 = 0$$

$$\alpha_c(A) = \alpha_c(s)\Big|_{s \to A} = A^2 + 2A + 5I$$

$$= \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 0 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 0 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 5 & 0 \\ 5 & 0 \end{pmatrix}$$

$$K = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 13 & 0 \\ 4 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 13 & 0 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} \underset{\downarrow}{4} & \underset{\downarrow}{5} \end{pmatrix}$$

$$\quad K_1 \qquad K_2$$

Report

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 1 & -2 \end{pmatrix} x + \begin{pmatrix} 1 \\ 2 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} x$$

~~Determine the feedback gain~~

Determine the feedback gain K to make

$\zeta = 0.5$ & & $t_s = 2$ sec

$t_s = \dfrac{4}{\zeta \omega_n}$ Calculate ω_n then

$S_{1,2} = -\zeta\omega_n \pm j\,\omega_n\sqrt{1-\zeta^2}$

$\zeta\omega_n = 2 \Rightarrow \omega_n = 4$

$S_{1,2} = -2 \pm j\,3.46$

Final Exam ⇒
- Root locus
- Bode Diagram
- Design lead & lag
- check Controllability
- Example محاضر اليوم